Technical Comments

Comment on "Radiative Ablation of Melting Solid"

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IN their recent paper, Prasad and Sinha[1] applied Biot's variational method to heat transfer of radiative ablation of melting solid and obtained expressions for the radiative surface temperature of the solid and melting distance in terms of time. Unfortunately, the results are in error. Apparently, the authors were not aware of some recent publications in this field. An earlier work by Chung and Yeh,[2] which had been published a year before the authors submitted their paper, has treated the same topic (except with the additional convective boundary condition) by using the same technique (except with the additional heat balance integral method). It has been shown in Ref. 2 that the predicted solidification rate and the total solidification time based on the approximate analyses are compared very well with numerical solutions of Goodling and Khader.[3] Recently, Yan and Huang[4] have developed a perturbation solution to the phase-change problem subject to convection and radiation. The results have been found to agree well with those of Ref. 2 when the perturbation parameter is small. Extensions of analysis in Ref. 2 based on Biot's method have also been made available.[5,6]

Attention is now given to the analysis of Prasad and Sinha. We feel that the following errors in Ref. 1 need to be corrected:

1) The last term on the left-hand side of Eq. (20) should be (20/3) $C\rho L$ instead of 60 $C\rho L$.

2) The second term on the right-hand side of Eq. (22) is 120ϕ instead of 40ϕ.

3) The general equation for dimensionless surface temperature given by Eq. (24) should be corrected to

$$\dot{\phi}(B_1\phi+B_2\phi^2+B_3\phi^3)/(1-\phi^4)^2 + \dot{\phi}(B_4\phi^5+B_5\phi^6+B_6\phi^7)/(1-\phi^4)^3 = A\phi+B\phi^2$$

where

$$B_1=120 \quad B_2=100\beta \quad B_3=25\beta^2\ldots$$

$$A=120 \quad B=40\beta$$

4) Equation (25) becomes invalid due to the errors involved in Eq. (24).

5) The limiting case of Eq. (24) with $L\to\infty$ is in turn in error. Equation (26) should be changed to

$$\frac{\dot{\phi}(1+3\phi^4)}{(1-\phi^4)^3}=1$$

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Table 1 Dimensionless surface temperatures and melting distance histories

ϕ	η	τ $\beta=0$	τ $\beta=5$
0.1	0.1000	0.1000	0.1125
0.2	0.2003	0.2004	0.2521
0.3	0.3024	0.3029	0.4228
0.4	0.4105	0.4127	0.6353
0.5	0.5333	0.5410	0.9152
0.6	0.6893	0.7135	1.3268
0.7	0.9212	0.9984	2.0524
0.8	1.3550	1.6494	3.7827
0.9	2.6170	4.5455	11.7617
1.0	∞	∞	∞

6) The solution of the above equation becomes completely different from that shown in Eq. (27). It can be represented by

$$\tau = \tfrac{3}{4}B_{\phi^4}(-1.2,-2)+\tfrac{1}{4}B_{\phi^4}(\tfrac{1}{4},-2)$$

where B_{ϕ^4} is incomplete beta function,[7] i.e.,

$$B_x(a,b)=\int_0^x t^{a-1}(1-t)^{b-1}\,dt$$

7) Both integrand and the upper bound on the right-hand side of Eq. (28) are in error. The corrected form should be

$$\tau=\int_0^\phi \{(B_1+B_2\phi+B_3\phi^2)/[(A+B\phi)(1-\phi^4)^2] + (B_4\phi^4+B_5\phi^5+B_6\phi^6)/[(A+B\phi)(1-\phi^4)^3]\}\,d\phi$$

The right-hand side of the above equation is integrated numerically using Simpson's rule and the double precision algorithm with $\Delta\phi=0.01$. The numerical results of ϕ and η with $\beta=0$ and 5 are summarized in Table 1. As can be seen the present results are quite different from those presented in Fig. 1 of Ref. 1.

References

[1] Prasad, A. and Sinha, S. N., "Radiative Ablation of Melting Solid," *AIAA Journal*, Vol. 14, Oct. 1976, pp. 1494-1497.

[2] Chung, B. T. F. and Yeh, L. T., "Solidification and Melting of Materials Subject to Convection and Radiation," *Journal of Spacecraft and Rockets*, Vol. 12, June 1975, pp. 329-333.

[3] Goodling, J. S. and Khader, M. S., "Inward Solidification with Radiation-Convection Boundary Condition," *Journal of Heat Transfer, Transactions ASME*, Vol. 96, Feb. 1974, pp. 114-115.

[4] Yan, M. M. and Huang, P. N. S., "Perturbation Solutions to Phase Change Problem Subject to Convection and Radiation," ASME Paper No. 77-WA/HT-16 presented at ASME Winter Annual Meeting 1977, Atlantic City, Ga.

[5] Yeh, L. T. and Chung, B. T. F., "Phase Change in a Radiating and Convective Slab with Variable Properties," *Journal of Spacecraft and Rockets*, Vol. 14, March 1977, pp. 178-182.

[6] Yeh, L. T. and Chung, B. T. F., "Transient Heat Conduction in A Finite Medium with a Phase Change," ASME Paper 76-WA/HT-3 1976 ASME Winter Annual Meeting, New York, N.Y.

[7] Erdelyi, A., Magnus, W., Oberhetlinger, F., and Tricomi, F., *Higher Transcendental Functions*, Vol. 1, McGraw-Hill, New York, 1953, p. 87.